A Game-Theoretic Model for Product Placement in Online Platform Markets ⋆

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Abstract: Motivated by online platforms such as Amazon, Airbnb, etc., we consider the following Bertrand game model of product placement: a number of sellers (e.g., apartment owners) are interested in placing their products on a platform's (e.g., Airbnb.com) website. We assume that the price of a product is determined by the the number of available sellers and their qualities, and the probability with which a platform user will buy a product is a function of the prices and the qualities, according to a multinomial logit model. In other words, the outcomes, i.e., the realized prices and sales, are determined by the Nash equilibrium of a Bertrand game. The platform can affect the outcome of the game by deciding on a mechanism to determine which products to display on their websites. For such a Bertrand game, we derive optimal mechanisms for the platform to maximize either social welfare or revenue.

Keywords: Online Platforms, Two-Sided Market, Display Control, Game Theory, Operations Research

1. INTRODUCTION

In recent years, we have witnessed the rise of many successful online platform markets, which have reshaped the economic landscape of the modern world. The online platforms facilitate the exchange of goods and services between buyers and sellers. For example, buyers can purchase goods from sellers on Amazon, eBay and Etsy, arrange accommodation from hosts on Airbnb and Expedia, and order transportation services from drivers on Uber and Lyft.

Compared with ancient markets, modern online platform markets have greater controls over price determination, search and discovery, information revelation, recommendation, etc. For example, Uber and Lyft adopt the *full control model*, in which the ride-sharing platforms use online matching algorithms to determine matches between drivers and riders as well as the fee for the route. Amazon and Airbnb use the *discriminatory control model*, where the platforms only control the list of products to display for each buyer's search, and the potential matches and transaction prices are determined by the preference of buyers and the competition among sellers. The rich control options for online platforms have led to an increasing discussion about the design of online marketplaces with different optimization objectives; see Banerjee et al. (2017); Arnosti et al. (2014); Kanoria and Saban (2017).

In this paper, we investigate the optimal social welfare and revenue under the discriminatory control model, in which the platform has only control over *search segmentation mechanisms - which products to display for each buyer's search*, and the transaction prices are endogenously determined by the competition among sellers under a Bertrand game model. Unlike traditional firms, most online platforms do not manufacture goods or provide services, and thus they also do not dictate the specific transaction prices. Instead, buyers and sellers jointly determine the prices at which the goods or services will be traded. For example, sellers set prices for their goods on Amazon, and hosts decide on the prices for their properties on Airbnb. These prices depend on the demand and supply for comparable goods and services in the market, and choosing different displayed products for buyers impacts the transaction prices and then the social welfare/revenue. Motivated by this, we study the role of search segmentation mechanisms in social welfare and revenue optimization in the discriminatory control model with endogenous prices.

2. MATHEMATICAL MODEL

We consider a two-sided market with n sellers $\mathbb{S} = \{1, 2, \cdots, n\}$ and one *representative* buyer. Each seller $i \in \mathbb{S}$ offers a product with quality θ_i and price p_i. We denote the quality and price vectors by $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_n)$ and $\boldsymbol{p} = (p_1, p_2, \cdots, p_n)$, respectively. The quality vector $\boldsymbol{\theta}$ is fixed, while the price vector $\boldsymbol{p}$ is determined by the competition among sellers. Without loss of generality, we assume the products' quality and prices are non-negative, *i.e.*, $\theta_i \geq 0$ and $p_i \geq 0$, and the sellers are sorted according to the product quality in a non-decreasing order, *i.e.*, $\theta_1 \geq \theta_2 \geq \cdots \geq \theta_n$. Given the quality $\boldsymbol{\theta}$ and prices $\boldsymbol{p}$ of all products, the buyer purchases one of the n products, or adopts an outside option, *i.e.*, buys nothing from this market. We normalize the problem parameters so that

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outside option's quality θ_0 and price p_0 are zero, *i.e.*, $\theta_0 = p_0 = 0$.

In the random utility model descried in McFadden (1986), the buyer derives utility u_i from purchasing the product $i \in \mathbb{S}$ or selecting the outside option $i = 0$ as follows

$$u_i \triangleq \theta_i + \xi_i - p_i, \tag{1}$$

where ξ_i is a random variable representing buyer's (private) preference about the ith alternative. Given the $n+1$ choices (n products and the outside option), the buyer selects the alternative with the maximum utility. Under the standard assumption that the random variables $\{\xi_i\}$ are independent and identically distributed (i.i.d.) with Gumbel distribution, Anderson et al. (1992) and McFadden (1974) have shown that the buyer selects $i \in \{0\} \cup \mathbb{S}$ with probability

$$q_i(\boldsymbol{p}) \triangleq Pr(u_i = \max_{j \in \{0\} \cup \mathbb{S}} u_j) = \frac{a_i}{1 + \sum_{j \in \mathbb{S}} a_j}, \tag{2}$$

where $a_i = \exp(\theta_i - p_i)$ for all $i \in \mathbb{S}$. We refer to q_i as the *demand* or *market share* of the alternative $i \in \{0\} \cup \mathbb{S}$. This choice model is known as the multinomial logit model in economic literature. We use $\boldsymbol{q} = (q_0, q_1, \cdots, q_n)$ to denote the market shares of all products.

Under the above model, we can also obtain an explicit form for the utility $\bar{u}$ of the representative buyer

$$\bar{u} \triangleq \mathbb{E}[\max_{i \in \{0\} \cup \mathbb{S}} u_i] = \log(1 + \sum_{i \in \mathbb{S}} a_i). \tag{3}$$

From the market share $q_i(\boldsymbol{p})$ in (2), we can express seller i's expected revenue $r_i(\boldsymbol{p})$ in terms of prices

$$r_i(\boldsymbol{p}) \triangleq p_i \times q_i(\boldsymbol{p}) = p_i \times \frac{a_i}{1 + \sum_{j \in \mathbb{S}} a_j}.$$

The social welfare of the two-sided market is measured by the sum of buyer's utility and the total revenue of sellers:

$$sw(\boldsymbol{p}) \triangleq \bar{u} + \sum_{i \in \mathbb{S}} r_i(\boldsymbol{p}) = \log(1 + \sum_{j \in \mathbb{S}} a_j) + \sum_{i \in \mathbb{S}} p_i \times \frac{a_i}{1 + \sum_{j \in \mathbb{S}} a_j}. \tag{4}$$

The revenue of the market is the total revenue of all sellers:

$$re(\boldsymbol{p}) \triangleq \sum_{i \in \mathbb{S}} r_i(\boldsymbol{p}) = \sum_{i \in \mathbb{S}} p_i \times \frac{a_i}{1 + \sum_{j \in \mathbb{S}} a_j}. \tag{5}$$

3. MAIN RESULTS

In this section, we first investigate the existence and uniqueness of equilibrium in the Bertrand game with a subset $S \subseteq \mathbb{S}$ sellers. In a Bertrand competition, seller $i \in S$ selects price p_i to maximize her revenue $r_i(\boldsymbol{p}) = p_i \times q_i(\boldsymbol{p})$, where the corresponding market share $q_i(\boldsymbol{p})$ is determined by the prices $\boldsymbol{p}$ of all products in (2). We can formally represent the Bertrand game as a triplet $G^b = (S, (\mathcal{P}_i)_{i \in S}, (r_i)_{i \in S})$, where S is a set of players, $\mathcal{P}_i$ is the strategy space of player $i \in S$ (*i.e.*, $\mathcal{P}_i \triangleq \{p_i | p_i \geq 0\}$), and $r_i(\boldsymbol{p})$ is the payoff of player $i \in S$. From Gallego et al. (2006), we have the following result.

Theorem 1. There exists a unique (pure) Nash equilibrium in the Bertrand game $G^b = (S, (\mathcal{P}_i)_{i \in S}, (r_i)_{i \in S})$. Furthermore, a vector of prices $\bar{\boldsymbol{p}} = (\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_n) \in \mathcal{P}$ satisfies $\partial r_i(\bar{\boldsymbol{p}})/\partial p_i = 0$ for all $i \in S$ if and only if $\bar{\boldsymbol{p}}$ is a Nash equilibrium in $\mathcal{P}$.

We can calculate a closed-form expression for the Nash equilibrium prices $\bar{\boldsymbol{p}}$ by solving the system of the first-order condition equations $\partial r_i(\bar{\boldsymbol{p}})/\partial p_i = 0$.

Theorem 2. In Bertrand game $G^b = (S, (\mathcal{P}_i)_{i \in S}, (r_i)_{i \in S})$, the Nash equilibrium price $\bar{p}_i$ and the corresponding market share $\bar{q}_i$ for each seller $i \in S$ are given by [1]

$$\bar{p}_i = \frac{1}{1 - V(\bar{q}_0 \times \exp(\theta_i - 1))} \text{ and } \bar{q}_i = V(\bar{q}_0 \times \exp(\theta_i - 1)),$$

where $\bar{q}_0$ is the unique solution to

$$\sum_{j \in S} V(\bar{q}_0 \times \exp(\theta_i - 1)) = 1 - \bar{q}_0. \tag{6}$$

With this result, we can obtain the equilibrium social welfare in the Bertrand game with the sellers $S \subseteq \mathbb{S}$

$$\overline{sw}(S) = -\log(\bar{q}_0) + \sum_{i \in S} \frac{\bar{q}_i}{1 - \bar{q}_i}. \tag{7}$$

Similarly, we can get the equilibrium revenue:

$$\overline{re}(S) = \sum_{i \in S} \frac{\bar{q}_i}{1 - \bar{q}_i}. \tag{8}$$

We next show how the choice of the set of sellers S can be optimized by the platform to maximize either social welfare or revenue. We have the following main results for social welfare maximization and revenue maximization.

Theorem 3. For social welfare maximization, the optimal mechanism is to display all products $\mathbb{S}$ in the platform.

Theorem 4. For revenue maximization, the optimal mechanism is to display the top k^* products, where k^* is determined by the quality of all products $\boldsymbol{\theta}$, and can be calculated in linear time.

The idea behind the proofs of these two results is to express the equilibrium social welfare/revenue in (7) and (8) as a function with an independent variable $\bar{q}_0$, and show certain properties of this function. To prove Theorem 3, we show that such function is decreasing with respective to $\bar{q}_0$, implying that adding a new product can always improve the equilibrium social welfare. To prove Theorem 4, we show that such function is quasi-convex, guaranteeing that the maximum revenue can be obtained at one of the two endpoints. These two endpoints correspond to the options of staying at the current set of products or involving a new product with the highest quality. With this critical observation, the platform will always select the available product with the highest quality when it decides to add a new product. Thus, if the current product set does not contain all the top k^* products, we can further improve the equilibrium revenue by repeatedly replacing one currently selected product with a new product with a higher quality. The detailed proofs can be found in Zheng and Srikant (2019).

We give a simple example to illustrate the difference between these two mechanisms. We consider two cases: a low quality case, *e.g.*, $\theta_1 = \theta_2 = \cdots = \theta_n = 0.5$, and a high quality case, *e.g.*, $\theta_1 = \theta_2 = \cdots = \theta_n = 10$. From Theorem 3, the optimal mechanisms for social welfare maximization in these two cases are to display all products. However, for revenue maximization, the platform still displays all products in the low quality case, but only displays the first product in the high quality case.

[1] For any $x \in (0, \infty)$, $V(x)$ is the solution $v \in (0, 1)$ satisfying $v \times \exp\left(\frac{v}{1-v}\right) = x$. This function is similar to the Lambert function $W(x)$, which we recall is the solution w satisfying $w \times exp(w) = x$.

REFERENCES

Anderson, S.P., De Palma, A., and Thisse, J.F. (1992). *Discrete choice theory of product differentiation.* MIT press.

Arnosti, N., Johari, R., and Kanoria, Y. (2014). Managing congestion in decentralized matching markets. In *EC*, 451–451.

Banerjee, S., Gollapudi, S., Kollias, K., and Munagala, K. (2017). Segmenting two-sided markets. In *WWW*, 63–72.

Gallego, G., Huh, W.T., Kang, W., and Phillips, R. (2006). Price competition with the attraction demand model: Existence of unique equilibrium and its stability. *Manufacturing & Service Operations Management*, 8(4), 359–375.

Kanoria, Y. and Saban, D. (2017). Facilitating the search for partners on matching platforms: Restricting agent actions. In *EC*, 117–117.

McFadden, D. (1974). Conditional logit analysis of qualitative choice behaviour. In P. Zarembka (ed.), *Frontiers in Econometrics*, 105–142. Academic Press New York, New York, NY, USA.

McFadden, D. (1986). The choice theory approach to market research. *Marketing Science*, 5(4), 275–297.

Zheng, Z. and Srikant, R. (2019). Optimal search segmentation mechanisms for online platform markets. Technical report. Available at `https://drive.google.com/open?id=18esA_BjxMvJbEkIvbdBruU1Ji5Am0KxJ`.